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Research Article

AN OVERVIEW: LINEAR PROGRAMMING PROBLEM AND IT'S APPLICATIONS

Sarita Patil and Sujata Albhar

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ABSTRACT

In mathematics, linear programming is a method of optimizing operations with certain constraints. A linear programming problem is a problem that deals with finding the optimal value of a given linear function. The optimal value can be either a maximum value or a minimum value. Here, the given linear function is considered as objective function. An objective function can have many variables, which are subject to conditions and have to satisfy a set of linear inequalities called linear constraints. Linear programming problems can be used to find optimal solutions for the following situations such as production problems, feeding problems, transportation problems, and allocation problems and so on. In this paper we have studied linear programming problems such as LPP, methods to solve and finally its applications.

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INTRODUCTION

A linear programming problem is a technique that helps in finding an optimal solution to a given problem modeled as a set of linear relationships. Linear programming problem is an optimization problem that determines the feasible region (area containing all possible solutions of the LPP) and optimizes the solution to obtain the maximum or minimum function value.

In simple words, a LPP is a collection of linear equation that is used to find the values of variables to optimize the objective function.

Basic Terminology

- **Decision Variable:** These are the unknown quantities that are expected to be estimated as outputs of the LPP solution.
- **Objective Function:** (in linear programming) a function that is to be maximized or minimized.
- **Constraints:** Constraints describe all the possible values that the variables of a linear programming problem require.
- **Feasible Solution:** A set of values of variables, which satisfy all constraints and all non-negativity constraints of variables, is called L.P.P. and is known as a Feasible Solution (F.S.).
- **Optimal Solution:** The solution that gives the maximum or minimum objective function value is the optimal solution.

Characteristics of LPP

There are six characteristics of the linear programming problem such as,

1. Constraints: Constraints should be expressed mathematically in terms of resources.
2. Objective Function: The objective function should be specified in a quantitative manner.
3. Linearity: The relationship between two or more variables in a function must be linear. This means the degree of the variable is one.
4. Finiteness: There should be finite and infinite number of inputs and outputs. If the function has infinite elements, an optimal solution is not possible.
5. Non-negativity: The variable value must be positive or zero. Its not be a negative value.
6. Decision Variable: The decision variable will determine the output. This gives the ultimate solution to the problem. For any problem, the first step is to identify the decision variables.

Methods to solve Linear Programming Problem

There are three methods to solve Linear Programming Method.

1. Graphical Method
2. Simplex Method
3. Two Phase Simplex Method

Now, we will discuss all methods of Linear Programming Problem

Graphical Method

A graphical method is used to optimize two-variable linear programming. If the problem has two decision variables, the graphical method is the best method to find the optimal solution. In this method, the set of inequalities is bounded. Then the inequality is plotted in the XY plane. Once, all the inequalities are plotted in the XY graph, the intersection region will help determine the feasible region. The feasible region will provide the optimal solution as well as explain what values our model can take.

Steps of Graphical Method

- Create an LPP.
- Create a graph and plot all limit lines.
- Determine the valid side of each limit line.
- Identify feasible solution areas.
- Find the optimal score.
- Calculate the coordinates of the optimal points.
- Evaluate the objective function at the optimum point.

Example: To Solve the given LPP by Graphical Method

Max $Z = 3X + 4Y$

Constraints: $X + Y \leq 4$

$X \geq 0$

$Y \geq 0$

Step 1:

The constraints written in a form of equations,

X	Y
0	4
4	0

Constraints: $X + Y = 4$

$X = 0$

$Y = 0$

Step 2

The constraints written in a form of equations,

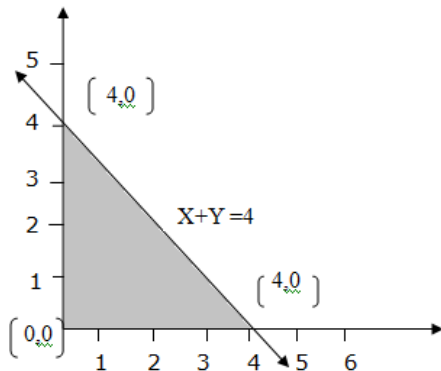
Constraints: $X + Y = 4$

Put $X = 0$, then $Y = 4$

Put $Y = 0$, then $X = 4$

Step 3 & 4

Plot the Graph and identify the feasible solution



The above shaded region is a feasible region of given LPP.

Step 5

Now, we find the value of the objective function at the corner points of the feasible region.

Corner Point	Corresponding Value of Z
(4,0)	12
(0,0)	0
(0,4)	16

Hence, maximum value of Z is 16 at the point (0, 4).

Simplex Method

The simplex method is one of the most popular methods for solving linear programming problems. It is an iterative process to obtain a feasible optimal solution. In this method, the value of the original variable is varied to obtain the maximum value for the objective function. The steps for linear programming simplex method are given below:

1. Convert the given general LPP into standard LPP

- Object function of LPP must be maximized. If it is to be minimized then we convert it into a problem of maximization by

$$Z_j - C_j = C_B X_i - C_j$$

- Check all the decision variable is greater than zero
- Express the problem in standard form by introducing slack and surplus variable to convert the inequality constraints into equations.
- All the values of right hand side must be positive.

2. Write the values of initial basic feasible solution.

3. Write the standard form LPP into matrix form.

4. Construct the initial simple table.

5. Calculate the value of $Z_j - C_j$ & check the basic feasible solution for optimality.

- If all $Z_j - C_j \geq 0$, the optimal solution will be obtained.
- If atleast one $Z_j - C_j$ is -ve then indicate by an arrow and this column is called key column.
- If more than one $Z_j - C_j$ is -ve then choose the most negative of them and this column is called key column.

6. Calculate Minimum Ratio,

$$\text{Mini Ratic} = \frac{X_B}{C_k}$$

$C_k = \text{Key Element, } > 0$

7. Construct the new simplex table by entering incoming vector.

8. Repeat step 5 & 6.

Slack Variable

If a constraint has \leq sign, then in order to make it equality, we have to add something positive variable to the left hand side.

Eg. $X_1 + X_2 \leq 2$ and $2X_1 + 4X_2 \leq 5$

Ans.- $X_1 + X_2 + S_1 = 2$ and $2X_1 + 4X_2 + S_2 = 5$

Surplus Variable

If a constraint has \geq sign, then in order to make it equality, we have to add something negative variable to the left hand side.

Eg. $X_1 + X_2 \geq 2$

$X_1 + X_2 - s_1 = 2$

Example: To Solve the given LPP by Simplex Method.

Max $Z = 3X_1 + 2X_2 + 5X_3$

Subject to $X_1 + 2X_2 + X_3 \leq 430$

$3X_1 + 2X_3 \leq 460$

$X_1 + 4X_2 \leq 420$

$X_1, X_2, X_3 \geq 0$

Step 1:

By introducing slack variable S_1, S_2, S_3
Convert the problem is standard form.

$$\begin{aligned} \text{Max } Z &= 3X_1 + 2X_2 + 5X_3 + 0S_1 + 0S_2 + 0S_3 \\ \text{Subject to } X_1 + 2X_2 + X_3 + S_1 &= 430 \\ 3X_1 + 2X_3 + S_2 &= 460 \\ X_1 + 4X_2 + S_3 &= 420 \\ X_1, X_2, X_3, S_1, S_2, S_3 &\geq 0 \end{aligned}$$

Step 2:

An initial basic feasible solution is given by,
 $X_1 = X_2 = X_3 = 0, S_1 = 430, S_2 = 460, S_3 = 420$

Step 3:

Writing in matrix form, $AX=B$

$$\begin{pmatrix} X_1 & X_2 & X_3 & S_1 & S_2 & S_3 \\ 1 & 2 & 1 & 1 & 0 & 0 \\ 3 & 0 & 2 & 0 & 1 & 0 \\ 1 & 4 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ S_1 \\ S_2 \\ S_3 \end{pmatrix} = \begin{pmatrix} 430 \\ 460 \\ 420 \end{pmatrix}$$

Step 4

An initial Simplex Table 1

C _B	B	C _j	3	2	5	0	0	0	Min Ratio
		X _B	X ₁	X ₂	X ₃	S ₁	S ₂	S ₃	
0	S ₁	430	1	2	1	1	0	0	430/1=430
0	S ₂	460	3	0	2	0	1	0	460/2=230
0	S ₃	420	1	4	0	0	0	1	-
Z _j -C _j			-3	-2	-5	0	0	0	

The Gray Shaded region is a Key Column and Key Rows
The Yellow shaded region is a Key Element

Calculation for above table

$$\begin{aligned} Z_j - C_j(X_1) &= C_B X_j - C_j = (0*1 + 0*3 + 0*1) - 3 = -3 \\ Z_j - C_j(X_2) &= C_B X_j - C_j = (0*2 + 0*0 + 0*4) - 2 = -2 \\ Z_j - C_j(X_3) &= C_B X_j - C_j = (0*1 + 0*2 + 0*0) - 5 = -5 \end{aligned}$$

In the above simplex table incoming vector is an X_3 and Outgoing Vector is a S_2

In the above table, all $Z_j - C_j$ does not obtain greater than zero values so we calculate another simplex table.

Simplex Table 2:

C _B	B	C _j	3	2	5	0	0	0	Min Ratio
		X _B	X ₁	X ₂	X ₃	S ₁	S ₂	S ₃	
0	S ₁	200	-1/2	2	0	1	-1/2	0	200/2=100
5	X ₃	230	3/2	0	1	0	1/2	0	-
0	S ₃	420	1	4	0	0	0	1	420/4=105
Z _j -C _j			9/2	-2	0	0	5/2	0	

In the above table incoming vector is an X_2 and Outgoing Vector is a S_1

The Gray Shaded region is a Key Column and Key Rows
The Yellow shaded region is a Key Element

Calculation for above table

- The key row is divided by Key Element in the above Simplex Table 1.

$$\frac{430*2 - 460*1}{2} = 200$$

$$\frac{420*2 - 460*0}{2} = 420$$

- 2

We have to calculate row S_1 or column X_1

$$X_1 = \frac{1*2 - 3*1}{2} = -\frac{1}{2}$$

We have to calculate row S_1 or column X_2

$$X_2 = \frac{2*2 - 2*1}{2} = 2$$

We have to calculate row S_1 or column S_2

$$S_2 = \frac{0*0 - 1*1}{2} = -\frac{1}{2}$$

we have to calculate row S_3 or column X_1

$$X_1 = \frac{1*2 - 3*0}{2} = 1$$

we have to calculate row S_3 or column X_2

$$X_2 = \frac{4*2 - 0*0}{2} = 4$$

we have to calculate row S_3 or column S_2

$$S_2 = \frac{0*2 - 1*0}{2} = 0$$

In the above table, all $Z_j - C_j$ does not obtain greater than zero values so we calculate another simplex table.

Simplex Table 3

C _B	B	C _j	3	2	5	0	0	0
		X _B	X ₁	X ₂	X ₃	S ₁	S ₂	S ₃
0	X ₂	100	-1/4	1	0	1/2	-1/2	0
5	X ₃	230	3/2	0	1	0	0	0
0	S ₃	20	2	0	0	0	-2	1
Z _j -C _j			4	0	0	1	2	0

Calculation for above table

- The key row is divided by Key Element in the above Simplex Table 2.

$$R_3 = R_3 - 4R_1$$

420	1	4	0	0	0	1
400	-1	4	0	2	-1	0
20	2	0	0	-2	1	1

Since all $Z_j - C_j \geq 0$, the solution is optimum given, $X_2 = 100, X_3 = 230, X_1 = 0$.

$$\begin{aligned} \text{Max } Z &= C_B X_B \\ &= (2*100 + 5*230 + 0*20) \\ &= 1350 \end{aligned}$$

Max Z = 1350

Application of Linear Programming Problem

Various domains of linear programming problems like manufacturing/ transportation/ sports/ share market/ engineering/ energy industry/ machine learning etc. in has their application. But here, we will discuss some of its real-life applications.

Let the number of tasks be 'n' and the number of people completing the task be 'm'. Now, assign tasks so that overall productivity is optimal. We find the optimal solution by determining the minimum cost, the time taken by each person or the maximum profit. This type of problem is a optimal assignment problem.

Manufacturing Industries: Industries use LPP models to maximize efficiency with minimum operating costs.

Transport Company: Companies like ola, uber and Rapido use LPP to optimize delivery routes. The objective of these companies is to reduce time and operational costs and increase revenue.

Machine Learning: Supervised learning works on the basics of linear programming.

CONCLUSION

Linear programming is a powerful technique for handling resource-allocation problems, cost-benefit-trade-off problems, and fixed-demand problems, as well as other problems with similar mathematical formulations. It has become an important

tool for numerous business and industrial organizations. In this article, we have discussed the Linear Programming Problem (LPP), methods of solving it and its application.

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